CHEMICAL ENGINEERING (GATE & PSUs)

Postal Correspondence

STUDY MATERIAL (Handwritten Notes)

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PROCESS DYNAMICS AND CONTROL

GATE-2022 Syllabus: Chemical Engineering: PDC

Measurement of process variables; sensors and transducers, P&ID equipment symbols, process modeling and linearization, transfer functions and dynamic responses of various systems, systems with inverse response, process reaction curve, controller modes (P, PI, and PID); control valves; transducer dynamics, analysis of closed loop systems including stability, frequency response, controller tuning, cascade and feed forward control.

PDC COURSE CONTENT

1. **Introduction**
2. **Modeling for Process Dynamics**
   - **Laplace Transform**
3. **Linear Open-Loop System**
   - **Forcing Function**
   - **First order system**
   - **Second order system**
4. **Linear Closed-Loop System**
   - **Process control**
   - **Types of controllers**
   - **Stability**
5. **Frequency Response**
   - **Bode stability criterion**
   - **Controller tuning**
   - **Advance control system (Cascade, Feed Forward control)**
   - **Inverse Response**
6. **Measurement of process variable**
   - **Control valve characteristics**
7. **Sensors , Transducers and P& ID equipment symbols**

Note for Student:

1. Full GATE Syllabus covers in Notes.

2. Total number of pages in PDC Notes = 384 Pages

3. No. of Questions solved in Notes = 175+ Questions

(GATE PYQs & other good quality question)

PROCESS DYNAMICS & CONTROL

OBJECTIVE OF A CHEMICAL PLANT :-

Raw material $\longrightarrow$ $\boxed{\text{PLANT}}$ $\longrightarrow$ product

Refinery

REQUIREMENTS :-

1) Safety (primary) : Reactor designed to operate within 10 kg/cm^2 pressure

2) Environmental Requirements : state & central environmental laws.

3) operational constraints :

Distillation column - flooding / foaming not take place

storage tank - overflow or go dry.

Heat exchanger - dry run condition

catalytic Reactor - temp. should not exceed upper limit (deactivation of catalyst occurs)

4) Economic : minimum operating cost, maximum profit.

5) Issues : (I). The influence of external disturbances

(II) The stability of a chemical process

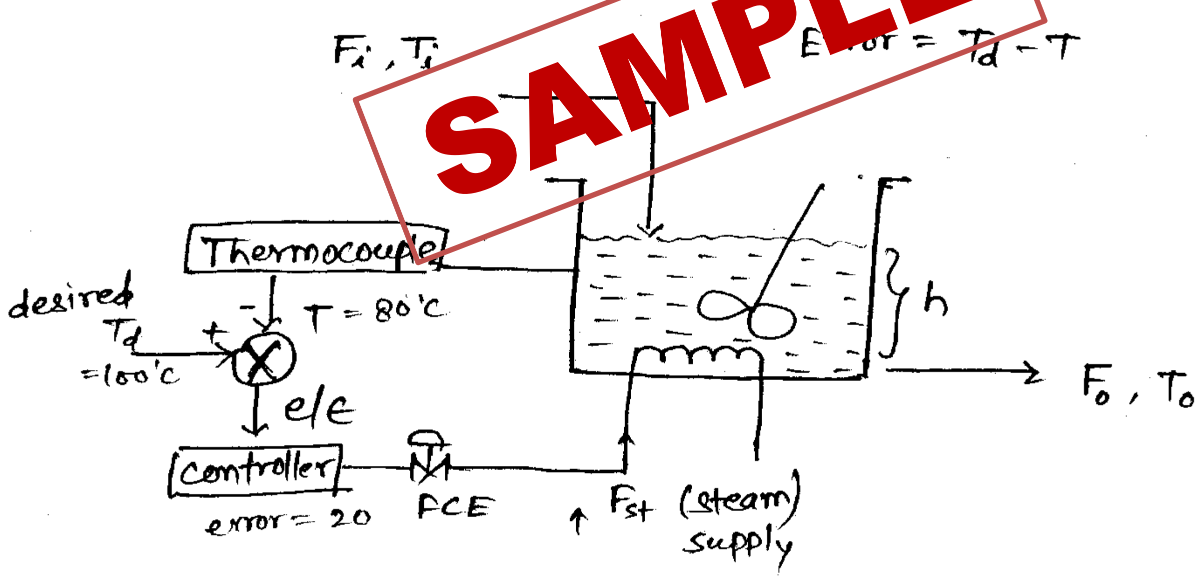
(III) The performance of a chemical process

* Basic aim of controller is to minimize the disturbances.

* <u>To meet</u> :-
  - product specification
  - Environmental Requirements
  - safety
  - operational constraint
  - Economics
  - Issues

  } control system is designed

* <u>control system</u> :-



* <u>objective</u> :- To maintain the temperature at its desired value ($T = T_d$) ($h = h_d$)

* <u>startup procedure of a process</u> :- If process remains in steady state all the time, no need of control variables.

LAPLACE TRANSFORM

- → Mathematical operator / tool
- → It helps to solve differential equation by converting them into algebraic equation.
- → Chemical processes are mathematically represented through a set of differential equations involving derivatives of process states.
- → consider a function $f(t)$,

then for all $\underline{t > 0}$

$$\left[L\{f(t)\} = F(s) = \int_0^\infty e^{-st} f(t)\, dt \right]$$

$\left(\begin{array}{l} s = a + ib \\ s \text{ is in complex plane} \end{array} \right)$

's' should be treated as a constant, when solving this integration.

* Some Basic formulas:

function — Laplace transform

1) $L\{1\} = 1/s$

2) $L\{k\} = k/s$

** 3) $\boxed{L\{t^n\} = \dfrac{n!}{s^{n+1}}}$

4) $L\{e^{-at}\} = \dfrac{1}{s+a}$

5) $L\{e^{at}\} = \dfrac{1}{s-a}$

* $L\{t\} = \dfrac{1}{s^2}$

6) $L\{\sin at\} = \dfrac{a}{s^2 + a^2}$

7) $L\{\cos at\} = \dfrac{s}{s^2 + a^2}$

8) $L\{\sinh at\} = \dfrac{a}{s^2 - a^2}$

9) $L\{\cosh at\} = \dfrac{s}{s^2 - a^2}$

3) Translation theorem :- (second shifting theorm)

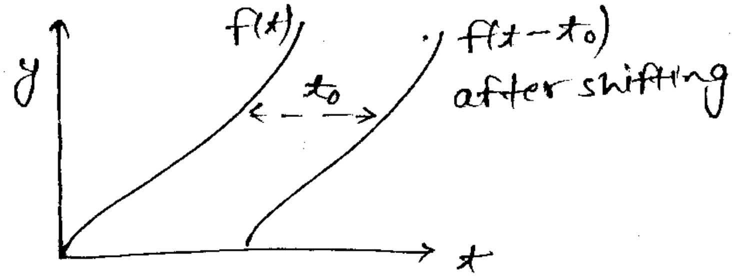
If $L\{f(t)\} = f(s)$

then

$$L\{f(t-t_0)\} = e^{-t_0 s} \cdot f(s)$$

$$L\{f(t) \cdot u(t-a)\} = e^{-as} L[f(t+a)]$$

$$L\{1 \cdot u(t-a)\} = \frac{e^{-as}}{s}$$



Imp.
4) Laplace transform of Derivatives :-

if $L\{f(t)\} = f(s)$, [f(0) is value of function at $t=0$]

① $L\left\{\frac{d^n}{dt^n} f(t)\right\} = s^n f(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \cdots - s^0 f^{(n-1)}(0)$

② $L\left\{\frac{d}{dt} f(t)\right\} = s f(s) - f(0)$

③ $L\left\{\frac{d^2}{dt^2} f(t)\right\} = s^2 f(s) - s f(0) - f'(0)$

*Note :- Differentiation w.r.t in time domain is equivalent to multiplication by s in s-domain.

Imp.
5) Laplace transform of Integrals :-

If $L\{f(t)\} = f(s)$

then

$$\left[L\left\{ \int_0^t f(t)\, dt \right\} = \frac{f(s)}{s} \right]$$

* Note :- Integration w.r.t to time in time domain is equivalent to division by s in s-domain.

Imp.
6) **Final value theorem :- [FVT]**

$$\boxed{\lim_{t\to\infty} f(t) = \lim_{s\to 0} s \cdot f(s)}$$

(Lt = Limit)

Imp
7) **Initial value theorem :- [IVT]**

$$\boxed{\lim_{t\to 0} f(t) = \lim_{s\to\infty} s \cdot f(s)}$$

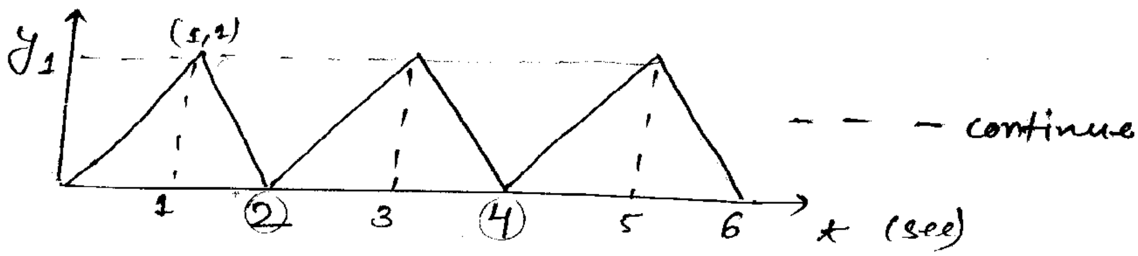
Laplace of periodic function :-

If a function as a periodic function with a time period T then the Laplace of that periodic function is given by, it means $f(t+T) = f(t)$

$$\left[F(s) = \frac{\int_0^T e^{-st} \cdot f(t)\, dt}{(1 - e^{-sT})} \right]$$

$f(t)$: Is the function which is repeating itself after time T.

Example



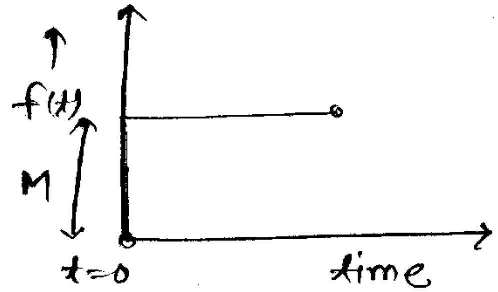
T = 2 sec

Standard input / Disturbance / Forcing function :→

(1) Step function :

$f(t)$ = deviation variable

$$\left.\begin{aligned} f(t) &= 0, \quad t < 0 \\ f(t) &= M, \quad t \geq 0 \end{aligned}\right\} f(t)$$

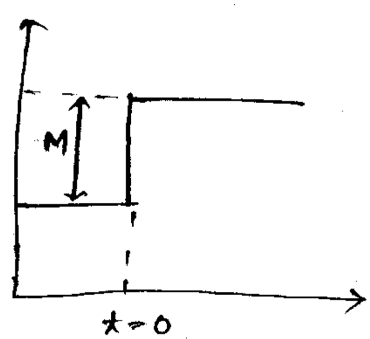


M = magnitude of step function

$$\boxed{L\{f(t)\} = \frac{M}{s}} \quad ; \quad t \geq 0$$

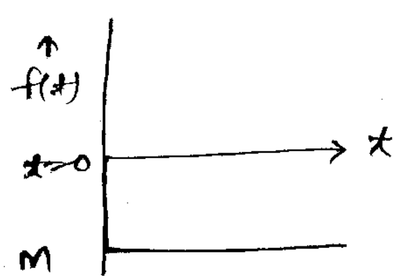
→ A step function of unit magnitude (M = 1) is denoted by $u(t)$ and known as "unit step function"

→ A step function of magnitude M can be denoted by $M\,u(t)$

* 

Also a step function (shifting of origin)

* unit step function $\boxed{L\{u(t)\} = \frac{1}{s}}$

* 

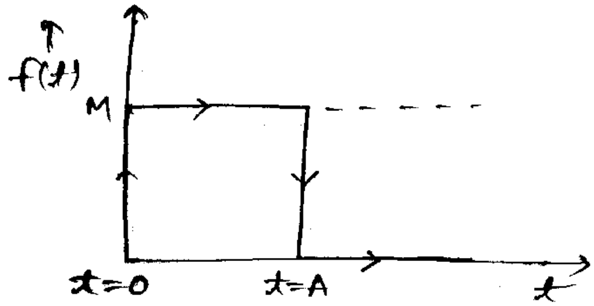
$f(t) = -M\,u(t)$

(2) Pulse function :- (Rectangular pulse function)

$f(t) = 0, \quad t < 0$

$f(t) = M, \quad 0 \le t \le A$

$f(t) = 0, \quad t > A$



$$L\{f(t)\} = \int_0^A M e^{-st}\, dt$$

$$= \frac{M e^{-st}}{(-s)}\Bigg|_0^A = \frac{M}{(-s)}\left(e^{-As} - 1\right)$$

$$\boxed{L\{f(t)\} = \frac{M}{s}\left(1 - e^{-As}\right)}$$

* pulse function = combination of two step function

$$\boxed{f(t) = M u(t) - M u(t-A)}$$

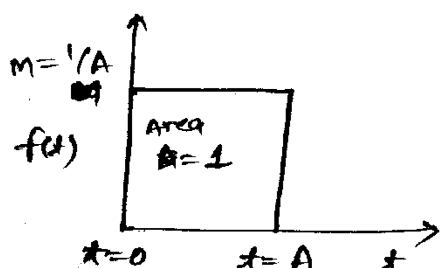
$$L\{f(t)\} = \frac{M}{s} - \frac{M e^{-As}}{s}$$

$$\boxed{L\{f(t)\} = \frac{M}{s}\left(1 - e^{-As}\right)}$$

* A unit pulse function is a pulse function for which the area becomes unity.

$$\left[L\{f(t)\} = \frac{1}{A \cdot s}\left(1 - e^{-As}\right) \right]$$

Area $= \frac{1}{A} \times A = 1 = \delta_A(t)$

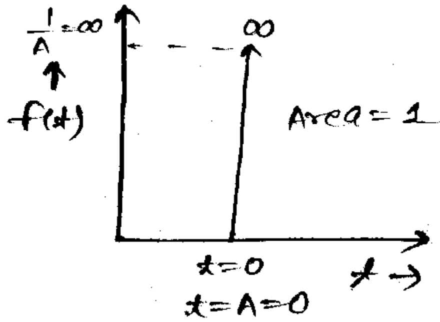


→ unit pulse function is represented by $\delta_A(t)$.

* Note :- The step function and pulse function are bounded in nature.

⟨3⟩ Impulse function :→

$$\boxed{f(t) = \delta(t)}$$



$\left(M = \frac{1}{A}\right)$

* Laplace of Impulse function :-

The unit impulse function is analogous to pulse function whose duration is shink to zero without loosing the strength. Hence area under impulse curve remains one.

$$f(t) = \begin{cases} \infty , & t = 0 \\ 0 , & t \neq 0 \end{cases} \qquad \lim_{A \to 0} \left(A \times \frac{1}{A} \right) = 1$$

* Dirac delta function = $\delta(t)$

$$\boxed{\int_{-\infty}^{\infty} \delta(t)\, dt = 1}$$

* Laplace of unit pulse function $= \frac{1}{s}\left(1 - e^{-As}\right)$

$$A \to 0 , \quad \frac{1}{A} \to \infty$$

$$L\{\delta(t)\} = \lim_{\left(A \to 0, \frac{1}{A} \to \infty\right)} \frac{\left(1 - e^{-As}\right)}{As} \Rightarrow \text{L- Hospital Rule}$$

$$= \lim_{A \to 0} \frac{0 - e^{-As}(-A)}{A} = \lim_{A \to 0} e^{-As}$$

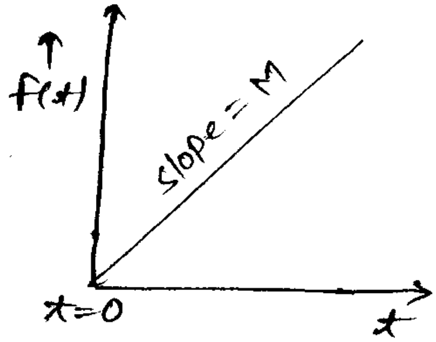
$$= e^{0} = 1$$

* $\boxed{L\{\delta(t)\} = 1}$; $L\{\text{Impulse function}\} = 1$

* $L\{A \text{ unit Impulse function}\} = A$
$(M = 1)$

(4) Ramp function :-

$$\left.\begin{array}{ll} f(t) = 0 & ; \ t < 0 \\ f(t) = Mt & ; \ t \geq 0 \end{array}\right\} f(t)$$



* Ramp input of slope M can be denoted by $M \underbrace{t\,u(t)}_{=1}$.

* Ramp input of slope 1 can be denoted by $t \cdot u(t)$

$$\boxed{L\{M t\, U(t)\} = \frac{M}{s^2}}$$

$$\left[L\{t\, u(t)\} = \frac{1}{s^2}\right]$$

* Input functions:-

		using FVT	Nature
1) step Input	$\boxed{F(s) = \frac{M}{s}}$	; $\lim_{s\to 0} s \cdot f(s) = \cancel{s} \times \frac{M}{\cancel{s}} = M$	Bounded
2) pulse Input	$\boxed{f(s) = \frac{M}{s}(1-e^{-As})}$	; $\lim_{s\to 0} s f(s) = \lim_{s\to 0} s \times \frac{M}{s}(1-e^{-As}) = M$	Bounded
3) Impulse Input	$\boxed{f(s) = 1}$	; $\lim_{s\to 0} s \cdot f(s) = s \times 1 = 0$	Bounded
4) Ramp Input	$\boxed{f(s) = \frac{M}{s^2}}$	; $\lim_{s\to 0} s \cdot f(s) = s \times \frac{M}{s^2} = \lim_{s\to 0} \frac{M}{s} = \frac{M}{0} = \infty$ (Infinity)	unBounded
5) sinusoidal Input $x(t) = A \sin\omega t$	$\boxed{f(s) = \frac{A \cdot \omega}{s^2+\omega^2}}$	; $= 0$	Bounded

Example of Input function :-

1) step Input :-  opening a valve suddenly and maintained them.

2) Ramp Input :-  filling of air ballon, ~~fell~~ switching on a room Heater.

3) pulse Input :-  open a valve suddenly & close after a finite time.

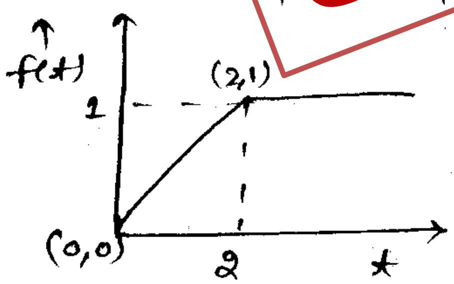
4) Impulse Input :- 

Hitting a cricket ball.

5) sinusoidal Input :- 

Atmospheric temperature.

Que:9) find the Laplace transform of following function



sol → $y = mx + c$ slope $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-0}{2-0} = \frac{1}{2}$

$f(t) = \frac{1}{2} t + c$, $c=0$ from $t=0$ to 2

at $\cancel{t=2}$, $f(t) = 1$, slope $m = 0$

from $t=2$ to ∞: $1 = 0 + c \Rightarrow \boxed{c=1} \Rightarrow f(t) = \frac{t}{2} + \cancel{1}$

Laplace of function $L\{f(t)\} = \int_0^{\infty} e^{-st} (f(t))\, dt$

$$= \int_0^{\infty} e^{-st}\left(\frac{t}{2} + 1\right) dt = \int_0^{2} e^{-st} \cdot \frac{t}{2}\, dt + \int_2^{\infty} e^{-st}\, dt$$

$\int I \cdot II\, dt = I \int II\, dt - \int \{\frac{d}{dt}(I) \cdot \int II\, dt\}\, dt$

ILATE

$$= \frac{1}{2}\left[t \cdot \frac{e^{-st}}{-s} - \int \frac{e^{-st}}{-s}\, dt\right]_0^2 + \frac{1}{-s}\left[e^{-\infty} - e^{-2s}\right]$$

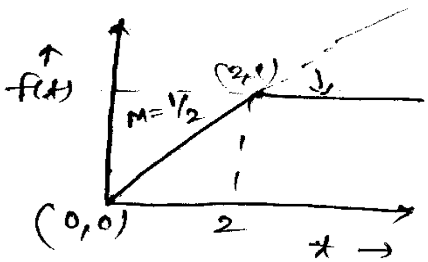
$$= \frac{1}{2}\left[t\cdot\frac{e^{-st}}{-s} - \int_0 \frac{e^{-st}}{-s}\,dt \right]_0^2 + \frac{1}{-s}\left[e^{-\infty} - e^{-2s}\right]$$

$$= \frac{1}{2}\left(-\frac{1}{s}\right)\left[t\,e^{-st} - \frac{e^{-st}}{-s} \right]_0^2 + \left(-\frac{1}{s}\right)\left[0 - e^{-2s}\right]$$

$$= -\cancel{\frac{e^{-2s}}{s}} - \frac{e^{-2s}}{2s^2} + \frac{1}{2s^2} + \cancel{\frac{e^{-2s}}{s}}$$

$$L\{f(t)\} = \underline{\frac{1}{2s^2}\left(1 - e^{-2s}\right)} \quad \text{Answer}$$

* **Shortcut :-**



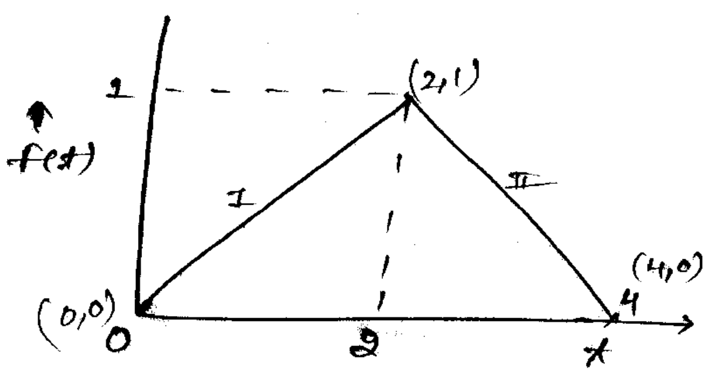
can be written as a sum of two ramp function

$$f(t) = \frac{1}{2}t\,u(t) - \frac{1}{2}t\,u(t-2)$$

$$L\{f(t)\} = \frac{1}{2s^2} - \frac{e^{-2s}}{2s^2}$$

$$L\{f(t)\} = \underline{\frac{1}{2s^2}\left(1 - e^{-2s}\right)} \quad \text{Answer}$$

Que→10) find the Laplace transform of following function



$$y = mx + c$$

Sol→ (I) slope $m_1 = \frac{1-0}{2-0} = \frac{1}{2}$

$f_1(t) = \frac{1}{2}t + c$

$c = 0$

$f_1(t) = t/2$

(II) slope $m_2 = \frac{0-1}{4-2} = -\frac{1}{2}$

$f_2(t) = -\frac{1}{2}t + c$

$0 = -\frac{4}{2} + c \Rightarrow c = +2$

$f_2(t) = -t/2 + 2$

PROCESS DYNAMICS

study of change in the output behaviour of a process when it is disturbed by a particular type of input.

Input $x(t)$ → Process → output $y(t)$

forcing function $x(t)$ → Model → Response $y(t)$

Mathematical model

* Model → To understand the process we write a model.
  → Mathematical model can be formed by writing the conservation equation on the 'control volume'.

* control volume :- In a control volume the value of important parameters should not change with position.

C_A, T ρ, P	control Volume

* Conservation Law's :-

(1) Mass Balance :- ⓐ overall Mass balance :

$$\left[\text{Mass in} - \text{Mass out} = \text{Accumulation}\right]$$

unit = mass/time

ⓑ Component mass balance :-

$$\left[\text{mass in} - \text{Mass out} + \text{mass generated} - \text{mass consumed} = \text{Accumuation}\right]$$

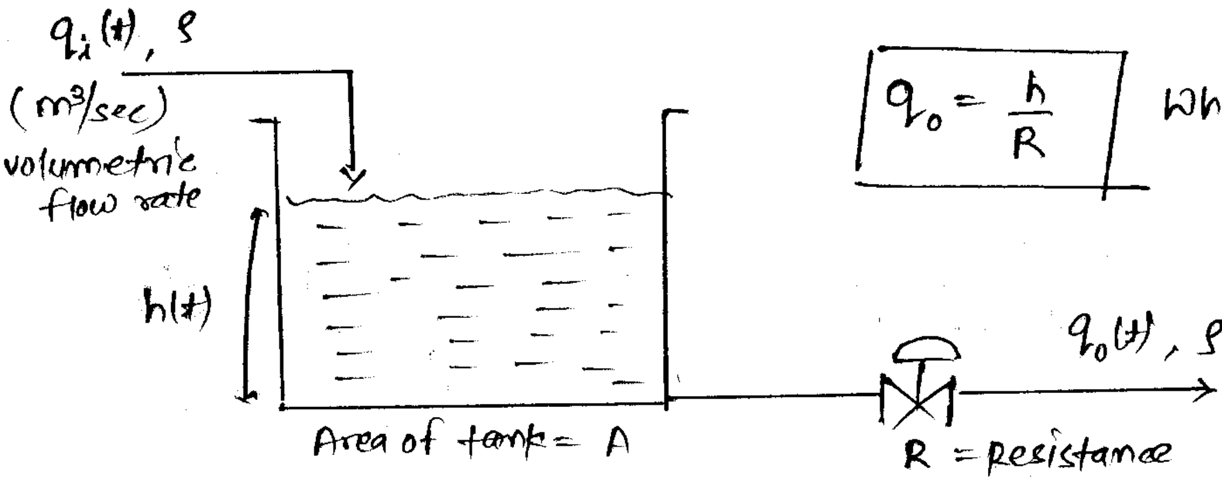
unit → mass/time

First order systems :- (Examples of 1st order system)

1) Liquid - Level system
2) Mercury thermometer system (Thermometer system)
3) Constant ~~to~~ temperature holdup system (Heating system)
4) constant concentration holdup system
5) pure capacitive process
6) Liquid mixing tank system (mixing process)

1) LIQUID LEVEL SYSTEM :- (Self Regulating system)

* Assumption :- (i) Density is constant
  (ii) Resistance is linear
  (iii) Lumped parameter (All resistance in FCE, valve)
  (iv) capacitance (storage) of mass only in tank.
  (v) temperature is constant



$$\boxed{q_o = \frac{h}{R}}$$ When R is linear

→ If all the resistance are working at one place, system is called first order system,

→ In our system R is only offered by the valve.

* overall mass balance:

$$\text{mass in} - \text{mass out} = \text{Accumulation}$$

mass flow rate $\quad q_i \rho - q_o \rho = \frac{d}{dt}(\rho \times A \times h)$

unit $\quad \frac{m^3}{s} \times \frac{kg}{m^3} = kg/sec \quad ; \quad \frac{1}{s}\left(\frac{kg}{m^3} \times m^2 \times m\right) = kg/sec$

$$m = \rho \times \text{volume}$$
$$m = \rho \times (A \times h)$$

$$\Rightarrow \quad q_i - q_o = A\frac{dh}{dt} \quad \text{—— (1)}$$

put $\boxed{q_o = \frac{h}{R}} \Rightarrow q_i - \frac{h}{R} = A\frac{dh}{dt} \quad \text{—— (2)}$

* conservation equation at steady state

$$q_{iss} - \frac{h_{ss}}{R} = A\frac{dh_{ss}}{dt} \to 0 \quad \text{—— (3)}$$

$$q_{iss} - \frac{h_{ss}}{R} = 0$$

at steady state $h_{ss} = \text{constant}$

$$\frac{dh_{ss}}{dt} = 0$$

Now, eqn (2) − eqn (3)

$$(q_i - q_{iss}) - \frac{(h - h_{ss})}{R} = A\frac{d}{dt}(h - h_{ss})$$

deviation variable $\quad Q_i = q_i - q_{iss}, \quad H = h - h_{ss},$

$$\boxed{Q_i(t) - \frac{H(t)}{R} = A\frac{dH}{dt}} \quad \text{—— (4)}$$

↳ equation in the form of deviation variables.

$$\Rightarrow \left[Q_i(t) - \frac{H(t)}{R} = A\frac{dH}{dt} \right] \quad \text{——— (4)}$$

taking Laplace transform

$$\Rightarrow Q_i(s) - \frac{H(s)}{R} = A\left[sH(s) - \cancel{H(0)} \right] \xrightarrow{} 0$$

$$\because H(0) = h(0) - h_{ss} \qquad \Big| \text{ at } t=0,\ h(0) = h_{ss}$$

$$H(0) = h_{ss} - h_{ss}$$

$$H(0) = 0$$

$$\Rightarrow Q_i(s) - \frac{H(s)}{R} = A \cdot sH(s)$$

$$\Rightarrow Q_i(s) = H(s)\left(\frac{1}{R} + As \right)$$

Gain, time constant

$$\Rightarrow \boxed{\frac{H(s)}{Q_i(s)} = \frac{R}{1+(AR)s}}, \quad K_p = R \quad \tau = AR \quad \text{——— (5)}$$

↳ Transfer function for the liquid level system relating change in liquid to the change in inlet flow rate.

$$\boxed{q_o = \frac{h}{R}} \Rightarrow q_{o_{ss}} = \frac{h_{ss}}{R}$$

$$\Rightarrow \boxed{Q_o(s) = \frac{H(s)}{R}} \Rightarrow H(s) = R \cdot Q_o(s)$$

$$\Rightarrow \text{put in eq}^n \text{ (5)} . \quad \frac{\cancel{R}\, Q_o(s)}{Q_i(s)} = \frac{\cancel{R}}{1+(AR)s}$$

$$\Rightarrow \boxed{\frac{Q_o(s)}{Q_i(s)} = \frac{1}{1+(AR)s}} \quad K_p = 1 \quad \tau = AR$$

Capsule summary of first order system :

SR. No.	system	T/F	K_p	τ_p
1)	Liquid Level system	$\boxed{\frac{H(s)}{Q(s)} = \frac{R}{AR\cdot s+1}}$;	R ,	AR
2)	Mercury thermometer system	$\boxed{\frac{T_o(s)}{T_I(s)} = \frac{1}{\frac{mC_p}{hA}\cdot s+1}}$;	1 ,	$\frac{mC_p}{hA}$
3)	Mixing process	$\boxed{\frac{C_{Ao}(s)}{C_I(s)} = \frac{1}{\frac{v}{q}\cdot s+1}}$	1 ,	$\frac{v}{q}$
4)	Heating system (Temp. dynamic)	$\boxed{\frac{T_o(s)}{w(s)} = \frac{1/q\rho C_p}{\frac{v}{q}s+1}}$;	$\frac{1}{q\rho C_p}$,	$\frac{v}{q}$
5)	Concentration system (1st order rxn)	$\boxed{\frac{C_{Ao}(s)}{C_{AI}(s)} = \frac{1/(1+kv/q)}{\frac{v/q}{(1+k\frac{v}{q})}\cdot s+1}}$	$\frac{1}{(1+kv/q)}$,	$\frac{v/q}{(1+kv/q)}$
6)	Pure capacitive process	$\boxed{\frac{H(s)}{q_I(s)} = \frac{1}{As}}$	$\frac{1}{A}$,	1

Standard transfer function for second order system :-

$$\boxed{G(s) = \frac{Y(s)}{X(s)} = \frac{k_p}{\tau^2 s^2 + 2\xi\tau s + 1}}$$

* **Note:-** That it required two parameters, τ and ξ to characterize the dynamics of a second order system in contrast to only one parameter (τ_p) for a first-order system.

$$\left[G(s) = \frac{Y(s)}{X(s)} = \frac{k_p}{\tau_p s + 1} \right]$$

Type of second order system :-

(1) Inherently second order system :- (self made second order system)

Example:- (I) viscous damper

(II) U-tube manometer → (GATE-2010)

(III) control valve / pneumatic valve

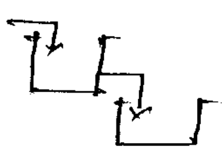
(IV) pressure transducers

→ Those system which on modeling gives second order differential equation known as second order system

(2) synthetic second order system :-

These are the system which are obtained by combining / Combination of two first order system in series.

Example : Multicapacity process



(a) Non-Interacting tank – critically damped ($\xi = 1$)

(b) Interacting tank → overdamped ($\xi > 1$)

Important terms for underdamped system : $\boxed{\xi < 1}$

→ The underdamped response occurs most frequently in control systems.

$$\left[Y(t) = 1 - \frac{1}{\sqrt{1-\xi^2}} e^{-\xi t/\tau} \sin(\omega t + \phi) \right]$$

$$\boxed{\omega = \frac{\sqrt{1-\xi^2}}{\tau}} \quad , \quad \boxed{\phi = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)}$$

*(1) **Overshoot** :-

It is a measure of how much the response exceeds the ultimate value following a step change.

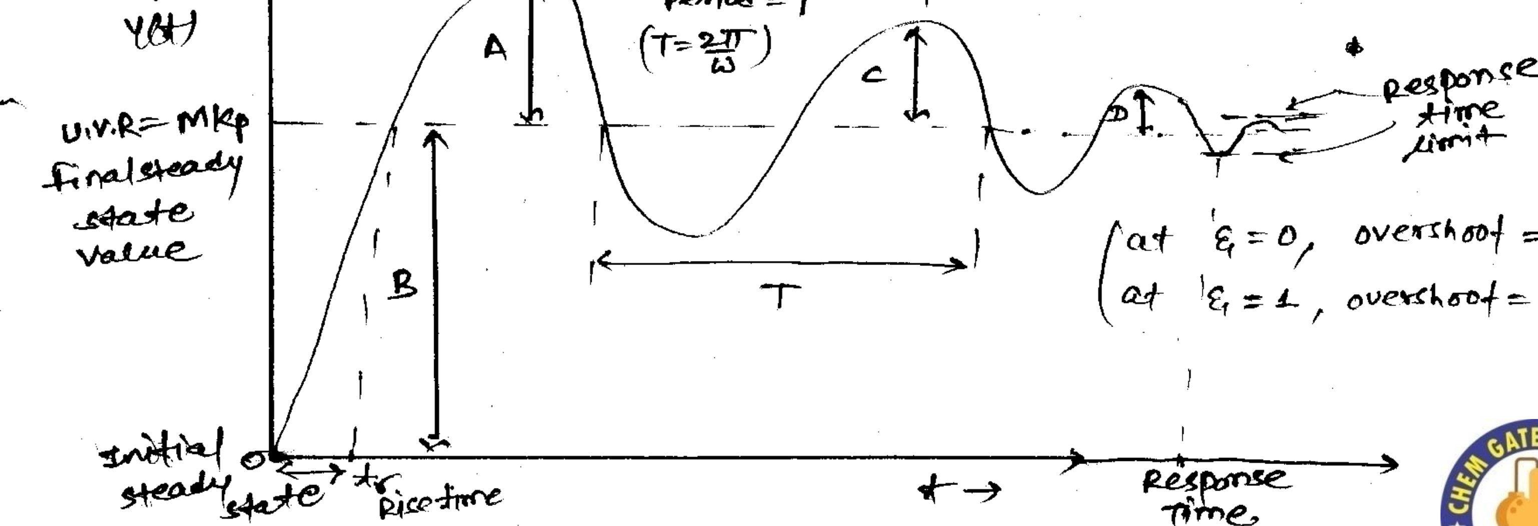
It is defined as the ratio A/B

where B is the ultimate value of Response and
A is the maximum amount by which the response exceeds its ultimate values.

$$\boxed{\text{overshoot} = \frac{A}{B} = \exp\left(\frac{-\pi\xi}{\sqrt{1-\xi^2}}\right)} = e^{-\pi\xi/\sqrt{1-\xi^2}}$$

* **Note**:- overshool should be as less as possible for stable system.

* **Note**:- $\boxed{\text{As } \xi \downarrow \ ; \ \text{overshoot} \uparrow}$

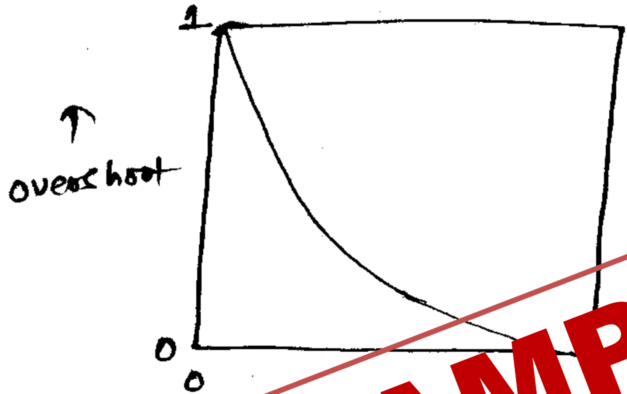


$$\left(\begin{array}{l} \text{at } \xi = 0, \ \text{overshoot} = 1 \\ \text{at } \xi = 1, \ \text{overshoot} = 0 \end{array} \right)$$

* Speed of Response

∴ $\left[1^{st} \text{ order} > \underset{(\xi < 1)}{\text{underdamped}} > \underset{(\xi = 1)}{\text{critical damped}} > \underset{(\xi > 1)}{\text{overdamped}} \right]$

* Overshoot $= \left[\dfrac{A}{B} = \exp\left(\dfrac{-\pi \xi}{\sqrt{1-\xi^2}} \right) \right]$ for underdamped ($\xi < 1$)



at $\xi = 0$, overshoot $= 1$

$\xi = 1$, overshoot $= 0$

$\boxed{\xi \downarrow, \text{ overshoot } \uparrow}$

system more unstable

oscillation ↑,

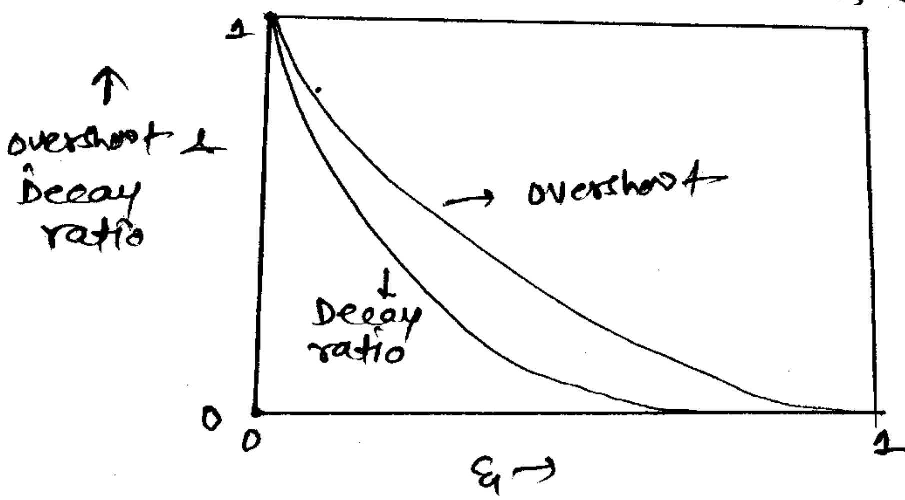
* (2) Decay Ratio :

It is defined as the ratio of the sizes of successive peaks and is given by C/A.

$$\left[\text{Decay Ratio} = \frac{C}{A} = \exp\left(\frac{-2\pi \xi}{\sqrt{1-\xi^2}} \right) = (\text{overshoot})^2 \right]$$

→ Larger ξ means greater damping, Hence greater decay

oscillation ↓, system become more stable



Numericals (second order system)

(Que-56) A step change of magnitude 4 is introduced into a system having the transfer function

$$G(s) = \frac{10}{s^2 + 1.6s + 4}$$

Calculate overshoot, Decay ratio & maximum value of Response, ultimate value of Response and period of oscillation.

Sol → I) Overshoot $= \frac{A}{B} = \exp\left(-\frac{\pi \xi}{\sqrt{1-\xi^2}}\right)$

$$G(s) = \frac{Y(s)}{X(s)} = \frac{10}{s^2 + 1.6s + 4}$$

$x(t) = 4$

$X(s) = 4/s$

$$Y(s) = \frac{4}{s}\left(\frac{10}{s^2 + 1.6s + 4}\right)$$

$$G(s) = \frac{10}{4\left(\frac{1}{4}s^2 + 0.4s + 1\right)} = \frac{K_p}{\tau^2 s^2 + 2\xi\tau s + 1}$$

$\tau^2 = \frac{1}{4} \Rightarrow \boxed{\tau = 0.5}$, & $2\xi\tau = 0.4$

$\tau = \frac{1}{2}$ $\qquad 2 \times \xi \times \frac{1}{2} = 0.4$

$\boxed{\xi = 0.4}$

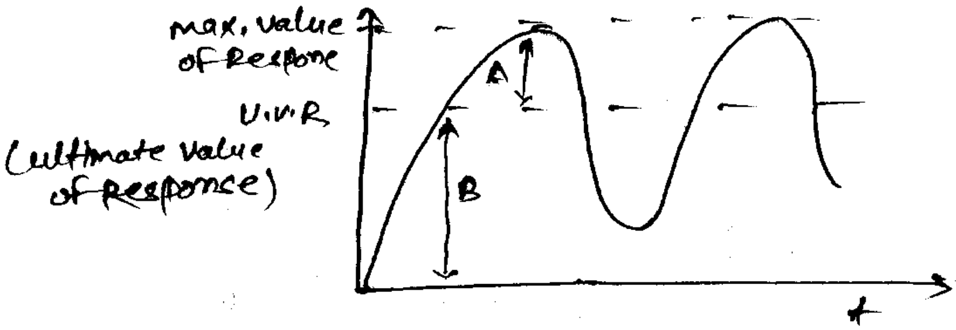
$$\text{Overshoot} = \exp\left(\frac{-\pi\xi}{\sqrt{1-\xi^2}}\right) = \exp\left(\frac{-\pi \times 0.4}{\sqrt{1-0.4^2}}\right) = 0.2538$$

$= \underline{0.254}$ Answer

II) Decay ratio $= \exp\left(-\frac{2\pi\xi}{\sqrt{1-\xi^2}}\right) = (\text{overshoot})^2$

$$= \exp\left(\frac{-2\pi \times 0.4}{\sqrt{1-0.4^2}}\right) \text{ or } = (0.2534)^2 = \underline{0.064} \text{ Answer}$$

III) Maximum value of Responese :-



* Maximum value of Response $= (A + B) = B\left(\frac{A}{B} + 1\right)$

Where B is ultimate value of Respone

$\frac{A}{B}$ is overshoot.

U.V.R by final value theorm $\boxed{\lim_{t\to\infty} y(t) = \lim_{s\to 0} s \cdot y(s)}$

$$Y(t) = \frac{4}{s} \cdot \frac{10}{s\left(\frac{1}{4}s^2 + 0.4s + 1\right)}$$

$$\lim_{t\to\infty} Y(t) = \lim_{s\to 0} s \times \frac{10}{s\left(\frac{1}{4}s^2 + 0.4s + 1\right)} = 10$$

U.V.R $= B = 10$

Overshoot $= 0.254$

Maximum value of Response $= A + B$

$= B\left(\frac{A}{B} + 1\right)$

$= 10\left(0.254 + 1\right)$

$= \underline{12.54}$ Answer

IV) Ultimate value of Response $= B$

U.V.R $= B = \underline{10}$ Answer