# CHEMICAL ENGINEERING (GATE & PSUs)

## Postal Correspondence

### STUDY MATERIAL (Handwritten Notes)

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# MASS TRANSFER

# GATE-2022 Syllabus: Chemical Engineering

Fick's laws, molecular diffusion in fluids, mass transfer coefficients, film, penetration and surface renewal theories; momentum, heat and mass transfer analogies; stage-wise and continuous contacting and stage efficiencies; HTU & NTU concepts; design and operation of equipment for distillation, absorption, leaching, liquid-liquid extraction, drying, humidification, dehumidification and adsorption., membrane separations (micro-filtration, ultrafiltration, nano-filtration and reverse osmosis).

# MASS TRNAFER COURSE CONTENT

1. **Introduction**
2. **Concept of Diffusion**
3. **Molecular Diffusion**
4. **Mass Transfer Coefficient**
5. **Distillation**
6. **Absorption**
7. **Humidification**
8. **Drying**
9. **Adsorption**
10. **Extraction**
11. **Membrane separation**

**Note for Student:**

**1. Full GATE Syllabus covers in Notes.**

**2. Total number of pages in MT Notes = 330 Pages**

**3. No. of Questions solved in Notes = 110+ Questions**

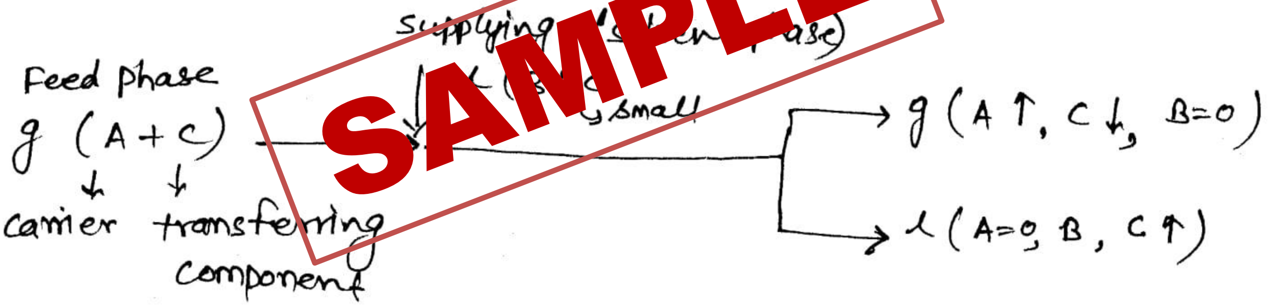
**( GATE PYQs & other good quality question)**

* The core separation processes in the chemical industry are - gas absorption and stripping, distillation, liquid - liquid and solid - liquid extraction, drying of a wet solid, adsorption, crystallization, and separation of multi-component mixtures.

All these separation processes involve mass transfer from one phase to another.

1) <u>Absorption</u> :- [Gas-Liquid contacting operation]

separation of a soluble species from a gas mixture by using a solvent is called "gas absorption".






$$\boxed{g \longrightarrow l}$$

gas → liquid

→ Transfer from g to l phase of solute

<u>Example</u> → separation of $CO_2$ from the ammonia synthesis gas using a solvent like aqueous ethanolamine.

2) <u>stripping or Desorption</u> : [Gas-Liquid contacting operation]

separation of a soluble species (absorbed) from a liquid mixture by using a solvent is called "stripping or desorption".

# * DIFFUSION *

→ Diffusion is the net movement of molecules or atoms from a region of high concentration to a reggion of low concentration as a result of random motion of the molecules or atoms.

→ Diffusion is driven by a gradient in chemical potential of the diffusing species.

## * Tyes of diffusion :-

1) Molecular diffusion

2) Turbulent or eddy diffusion

## (1) Molecular Diffusion → (In gases)



If the bulk fluid is stationary or moving in laminar motion in a direction normal to the concentration gradient the process is known as "Molecular diffusion".

→ Molecular diffusion is concerned with the movement of individual molecules through a substance by virtue of their thermal energy (kinetic theory of gases)

Example → Evaporation of napthalene ball in stationary air medium.

Diffusion →
- * Molecular diffusion
- * Mass transfer coefficient (M.T.C)
- * Interphase mass transfer

# Flux :⇒ (Amount of quantity passing through unit area per unit time)

The flux is defined as the rate of transport of species $i$ per unit area in a direction normal to the transport. The flux is calculated with respect to a fixed reference frame.

" The Net rate at which a component in a solution passes through unit area which is normal to the direction of diffusion in unit time ".

$$\text{flux} \rightarrow \underbrace{\frac{kg}{m^2\text{-}s}}_{\text{Mass flux}} \quad \text{or} \quad \underbrace{\frac{kmol}{m^2\text{-}s}}_{\text{Molar flux}}$$



## MASS flux $\left(\frac{kg}{m^2\text{-}s}\right)$

→ we will denote mass flux by small latters.

1) $n_i$ = mass flux of component $i$ w.r.t stationary observer

$$\text{mass flux} = \rho \times u$$

$$= \frac{kg}{m^3} \times \frac{m}{s} \Rightarrow \frac{kg}{m^2\text{-}s}$$

$$\boxed{n_i = \rho_i (u_i - 0)}$$

Where

$u_i$ = velocity of component $i$ w.r.t stationary observer

## MOLAR flux $\left(\frac{kmol}{m^2\text{-}s}\right)$

→ we will denote molar flux by capital letters.

1) $N_i$ = Molar flux of component $i$ w.r.t stationary observer

$$\text{molar flux} = c \times u$$

$$= \frac{kmol}{m^3} \times \frac{m}{s} \Rightarrow \frac{kmol}{m^2\text{-}s}$$

$$\boxed{N_i = c_i \times (u_i - 0)}$$

2) $i_i$ = mass flux of component $i$ when the observer is moving with mass average velocity

$$\boxed{i_i = \rho_i (u_i - U)}$$

where

$U \rightarrow$ mass average velocity

2) $I_i$ = Molar flux of component $i$ w.r.t the observer is moving with mass average velocity

$$\boxed{I_i = C_i (u_i - U)}$$

$U \rightarrow$ mass average velocity

3) $j_i$ = mass flux of component $i$ when the observer is moving with molar average velocity

$$\boxed{j_i = \rho_i (U_i - U)}$$

where

$U$ = molar average velocity

3) $J_i$ = molar flux of component $i$ when the observer is moving with molar average velocity

$$\boxed{J_i = C_i (U_i - U)}$$

SAMPLE

(molar flux - mass avg. velocity)

# $I_{total} = \Sigma I_i$

$= \Sigma C_i (U_i - U)$ → mass avg

$= C \dfrac{\Sigma C_i U_i}{C} - \Sigma C_i U$

$= CU - CU$

$$\boxed{I_{total} = C(U - U)}$$

$I_{total} \neq 0$

$U$ = mass avg. velocity
$U$ = Molar avg. velocity

Mass avg. velocity $\left(U = \frac{1}{\rho} \Sigma \rho_i u_i\right)$

Molar avg. velocity $\left(U = \frac{1}{C} \Sigma C_i u_i\right)$

$(\Sigma C_i = C)$

# $i_{total} = \sum i_i$ (Mass flux – mass avg. velocity)

$$= \sum \rho_i (u_i - u) \qquad u \to \text{mass avg velocity}$$

$$= \rho \frac{\sum \rho_i u_i}{\rho} - \sum \rho_i u \qquad \left( u = \frac{\sum \rho_i u_i}{\rho} \right)$$

$$= \rho u - \rho u \qquad \left( \sum \rho_i = \rho \right)$$

$$= 0$$

$$\boxed{i_{total} = 0}$$

# $J_{total} = \sum J_i$ (molar flux – molar avg. velocity)

$$= \sum C_i (U_i - U) \quad ; \; U \to \text{molar avg. velocity}$$

$$= C \frac{\sum C_i u_i}{C} - \sum C_i U \qquad \left( U = \frac{\sum C_i u_i}{C} \right)$$

$$= CU - U \sum C_i \qquad \left( \sum C_i = C \right)$$

$$= CU - CU$$

$$= 0$$

$$\boxed{J_{total} = 0}$$

# $- j_{total} = \sum j_i$ (mass flux – molar avg. velocity)

$$= \sum \rho_i (U_i - U)$$

$$= \rho \frac{\sum \rho_i u_i}{\rho} - \sum \rho_i U \qquad u \to \text{mass avg. velocity} \quad \left( u = \frac{\sum \rho_i u_i}{\rho} \right)$$

$$= \rho u - U \sum \rho_i$$

$$= \rho u - U \rho$$

$$j_{total} = \rho (u - U)$$

$$\boxed{j_{total} \neq 0}$$

Imp

# Flux for a Binary mixture :-

| Mass flux | Molar flux |
|---|---|
| 1) $n_1 + n_2 \neq 0$ | 1) $N_1 + N_2 \neq 0$ |
| 2) $\boxed{i_1 + i_2 = 0}$ | 2) $I_1 + I_2 \neq 0$ |
| 3) $j_1 + j_2 \neq 0$ | 3) $\boxed{J_1 + J_2 = 0}$ |
| (Mass flux) | (Molar flux) |

# Fick's first Law of Diffusion



" The Molar flux of species 'A' with respect to an observer moving with molar average velocity in a particular direction is directly proportional to the concentration gradient that exists in that direction.

$$J_{Az} \propto \frac{dC_A}{dz}$$

{ Here I am assuming one-dimensional diffusion is taking place in z-direction }

flux

$$\boxed{J_{Az} = -D_{AB} \frac{dC_A}{dz}}$$

→ fick's first law in 1-D cartesian coordinates

Radial diffusion

$$\left[ J_{Az} = -D_{AB} \frac{dC_A}{dr} \right]$$

In 1-D cylindrical coordinates

where $D_{AB}$ is proportionality constant

or

$D_{AB}$ = diffusivity, diffusivity coefficient

$D_{AB} \rightarrow$ Diffusivity of species A in B

It tellus us how easily A can diffuse into B

$$\boxed{J_{Az} = -D_{AB} \frac{dC_A}{dz}}$$

* Negative sign :- It denote that diffusion occurs in the direction of a drop in concentration.

* unit of diffusivity :- $D_{AB} = J \times \frac{dz}{dC_A}$

$$= \frac{kmol}{m^2\text{-}s} \times \frac{m}{kmol/m^3}$$

$$= m^2/s$$

$$\boxed{D_{AB} \rightarrow m^2/s}$$

SAMPLE

1) Mass transfer (Fick's law)

$$\left[ J_{Az} = -D_{AB} \frac{dC_A}{dz} \right] ; \quad \frac{dC_A}{dz} = \text{concentration gradient}$$

2) Heat transfer (fourier law)

$$\left[ q = -k \frac{dT}{dx} \right] ; \quad \frac{dT}{dx} = \text{Temperature gradient}$$

3) Fluid Mechanics (Newton's law)

$$\left[ \tau = -\mu \frac{du}{dy} \right] ; \quad \frac{dv}{dy} = \text{velocity gradient}$$

(1) fick's law (MT) :- $J_A = -D_{AB} \frac{dC_A}{dz}$

$$\text{flux of moles} = -\underline{\text{molar Diffusivity}} \times \frac{d}{dz}\left(\frac{mol}{m^3}\right)$$

(2) fourier law (HT) :-

$$\text{flux of Heat} \Rightarrow q = -\frac{k}{\rho C_p} \frac{dT}{dx} \rho C_p$$

$\left(\alpha = \frac{k}{\rho C_p}\right)$

$$q = -\alpha \frac{d}{dx}\left(\frac{heat}{m^3}\right)$$

$$q = \underline{\text{thermal/Heat Diffusivity}} \times \frac{d}{dx}\left(\frac{heat}{m^3}\right)$$



(3) Newton's Law (FM) :-

$$\text{flux of momentum} \Rightarrow \tau = -\frac{\mu}{\rho} \frac{du}{dy} \rho$$

$\left(\nu = \frac{\mu}{\rho}\right)$

$$\tau = -\nu \frac{d(\rho u)}{dy}$$

$$\tau = \text{momentum Diffusivity} \times \frac{d}{dy}\left(\frac{momentum}{m^3}\right)$$

$$\boxed{\text{flux of a quantity} = \text{diffusivity of that quantity} \times \frac{d}{dz}\left(\frac{quantity}{m^3}\right)}$$

Net flux $\Rightarrow (N = N_A + N_B)$

# Fundamental equation of M.T :-

Let us consider Binary mixture of A & B

$$J_A = -D_{AB}\frac{dC_A}{dz}$$

$$J_A = C_A(U_A - U)$$

$U$ = molar avg. velocity

$$U = \frac{\sum C_i u_i}{C}$$

$$J_A = C_A U_A - C_A U$$

$$J_A = C_A U_A - C_A\left\{\frac{C_A u_A + C_B u_B}{C}\right\}$$

$$J_A = N_A - \frac{C_A}{C}\left\{N_A + N_B\right\}$$

flux $N_A = C_A u_A$ ; $N_B = C_B u_B$

$$\boxed{N_A = J_A + \frac{C_A}{C}(N_A + N_B)} \quad \text{—— (1)}$$



similarly,

$$\boxed{N_B = J_B + \frac{C_B}{C}(N_A + N_B)} \quad \text{—— (2)}$$

eq$^n$ (1) & (2) are fundamental equations of Mass transfer.

Add eq$^n$ (1) & (2)

$$(N_A + N_B) = (J_A + J_B) + (N_A + N_B)\left(\cancel{\frac{C_A + C_B}{C}}\right)$$

$$C_A + C_B = C$$

$$\boxed{J_A + J_B = 0}$$

$$\boxed{J_A = -J_B}$$

if Assume $\boxed{C_A + C_B = C}$ constant

then $-D_{AB}\frac{dC_A}{dz} = -\left\{-D_{BA}\frac{dC_B}{dz}\right\}$ —— (3)

* If we Assume total concentration $c$ to be constant :-

$$C_A + C_B = C$$

$$\Rightarrow \frac{dC_A}{dz} + \frac{dC_B}{dz} = 0$$

$$\Rightarrow \left( \frac{dC_A}{dz} = -\frac{dC_B}{dz} \right) \quad \text{—— (4)}$$

from eq$^n$ (3) & (4)

$$-D_{AB}\left(-\frac{dC_B}{dz}\right) = D_{BA}\frac{dC_B}{dz}$$

$$\boxed{D_{AB} = D_{BA}}$$ if total conc$^n$. are constant

otherwise in general

$$\boxed{D_{ij} \neq D_{ji}}$$



* Total flux $\boxed{N_A = J_A + x_A N}$

$N_A \Rightarrow$ flux w.r.t stationary observer

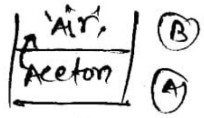
$J_A \rightarrow$ contribution of a molecular diffusion

$x_A N \rightarrow$ contribution of bulk flow

# Types of Molecular Diffusion ⇒ (steady-state)

1) Diffusion of A through Non-diffusing B

Example → (I) Evaporation of Napthalene (A) through stagnant Air (B)

(II) 


(III) Ammonia (A) were being absored form air (B) into water

2) Equimolar counter diffusion

Example → (I) Binary distillation $\left\{ \begin{array}{l} \text{Benzene (A)} \\ \text{toluene (B)} \end{array} \right\}$ ~~Toluene Tol~~

→ If vapourization of Heat are similiar (Approx)

(II) combustion . $C + O_2 \rightarrow CO_2$ complete combustion

for 1 mole C use 1 mole $O_2$ and produce $CO_2$

$\left\{ C + \frac{1}{2} O_2 \longrightarrow CO_2 \right.$ Not equimolar diffusion $\left. \right\}$



* (1) Diffusion of A through non-diffusing B ⇒ (ADNDB)

only napthalene diffuses $\quad N_A \neq 0, \quad N_B = 0$

fundamental equation of MT

$$\left[ N_A = J_A + \frac{C_A}{C} (N_A + N_B) . \right]$$

$N_A$ = constant , $N_B = 0$

$$\boxed{\frac{N_A}{N_A + N_B} = 1}$$

$N = N_A + N_B$

$N = N_A$

$$\boxed{N_A = J_A + \frac{C_A}{C}\left(N_A + \cancel{N_B}^{\,0}\right)}$$

$$N_A = J_A + \frac{C_A}{C}(N_A)$$

$$\boxed{N_A = J_A + x_A N_A}$$

$$N_A = \left(-D_{AB}\frac{dC_A}{dz}\right) + x_A N_A$$

$$N_A - x_A N_A = -D_{AB}\frac{dC_A}{dz}$$

$$\left[N_A(1-x_A) = -D_{AB}\frac{dC_A}{dz}\right] ✓$$

* Assume total concentration is constant

$$x_A = \frac{C_A}{C} \Rightarrow C_A = C x_A$$

$$dC_A = C\, dx_A \quad ✓$$



$$N_A(1-x_A) = -D_{AB}\cdot C\frac{dx_A}{dz}$$

$$\int_{z_1}^{z_2} dz = \frac{-D_{AB}\cdot C}{N_A}\int_{x_{A1}}^{x_{A2}}\frac{dx_A}{1-x_A}$$

$$(z_2 - z_1) = \cancel{(-)}\frac{D_{AB}\, C}{N_A}\cancel{(-)}\ln\left(\frac{1-x_{A2}}{1-x_{A1}}\right)$$

Limit at $z = z_1,\quad x_A = x_{A1}$

$z = z_2,\quad x_A = x_{A2}$

$$\Rightarrow \left[N_A = \frac{D_{AB}\cdot C}{(z_2 - z_1)}\ln\left(\frac{1-x_{A2}}{1-x_{A1}}\right)\right] \text{———} (1)$$

Mole fraction at location 1 & 2

$x_{A_1} + x_{B_1} = 1 \Rightarrow 1 - x_{A_1} = x_{B_1}$

$x_{A_2} + x_{B_2} = 1 \Rightarrow 1 - x_{A_2} = x_{B_2}$

$x_{A_1} + x_{B_1} = x_{A_2} + x_{B_2}$

$$\boxed{x_{A_1} - x_{A_2} = x_{B_2} - x_{B_1}}$$

$$N_A = \frac{D_{AB} \cdot C}{(Z_2 - Z_1)} \ln\left(\frac{x_{B_2}}{x_{B_1}}\right) \cdot \frac{(x_{A_1} - x_{A_2})}{(x_{A_1} - x_{A_2})}$$

$$N_A = \frac{D_{AB} \cdot C}{z} \ln\left(\frac{x_{B_2}}{x_{B_1}}\right) \times \left(\frac{x_{A_1} - x_{A_2}}{x_{B_2} - x_{B_1}}\right)$$

$$\left[ x_{Blm} = \frac{x_{B_2} - x_{B_1}}{\ln x_{B_2}/x_{B_1}} \right]$$

$x_{Blm}$ = logarthmic mean of $x_B$ b/w position 1 & 2.



Imp.

$$\boxed{N_A = \frac{D_{AB} \cdot C}{z} \frac{(x_{A_1} - x_{A_2})}{x_{Blm}}} \quad \text{--- (1)}$$

* find relation b/w $x_{Blm}$ & $C_{Blm}$ :-

$C_A + C_B = C$

$x_A = \frac{C_A}{C}$ | $x_B = \frac{C_B}{C}$

$x_A C_A = C_A$ | $x_B \cdot C = C_B$

$$x_{Blm} = \frac{x_{B_2} - x_{B_1}}{\ln(x_{B_2}/x_{B_1})}$$

$$x_{Blm} \cdot C = \frac{(x_{B_2} - x_{B_1}) \cdot C}{\ln(x_{B_2}/x_{B_1} \cdot C/C)}$$

$$C \cdot x_{Blm} = \frac{C_{B_2} - C_{B_1}}{\ln C_{B_2}/C_{B_1}} \Rightarrow \boxed{x_{Blm} = \frac{C_{Blm}}{C}}$$

* **for gases** :-

$$\left\{\begin{array}{l} x_{A1} \rightarrow y_{A1} \\ x_{A2} \rightarrow y_{A2} \\ x_{Blm} \rightarrow y_{Blm} \\ C \rightarrow P/RT \\ C_{A1} \rightarrow P_{A1}/RT \end{array}\right\} \quad \left(y_{A1} = \frac{P_{A1}}{P}\right) \quad \left(y_{Blm} = \frac{P_{Blm}}{P}\right)$$

$\Rightarrow$

$$N_A = \frac{D_{AB} \cdot C}{z} \frac{(x_{A1} - x_{A2})}{x_{Blm}}$$

$$x_{Blm} = C_{Blm}/C$$

$$N_A = \frac{D_{AB}}{z} \frac{(C_{A1} - C_{A2})}{C_{Blm}/C}$$

$$C_{A1} = C x_{A1}$$

$$C_{A2} = C x_{A2}$$

$$\boxed{N_A = \frac{D_{AB} \cdot C}{z} \frac{(C_{A1} - C_{A2})}{C_{Blm}}} \quad \text{--- (2)}$$

$$\frac{C_A}{C} = y_A$$

$$N_A = \frac{D_{AB} \, P/RT}{z} \left(\frac{y_{A1} - y_{A2}}{y_{Blm}}\right)$$

$$P y_{A1} = P_{A1}$$

$$P y_{A2} = P_{A2}$$

$$C = P/RT$$



or

$$N_A = \frac{D_{AB}}{RTz} \frac{(P_{A1} - P_{A2})}{P_{Blm}/P}$$

Imp.

$$\boxed{N_A = \frac{D_{AB} \cdot P}{RTz} \frac{(P_{A1} - P_{A2})}{P_{Blm}}} \quad \text{--- (3)}$$

Imp

$$\boxed{N_A = \frac{D_{AB} P}{RTz} \ln\left(\frac{P - P_{A2}}{P - P_{A1}}\right)} \quad \text{--- (4)}$$

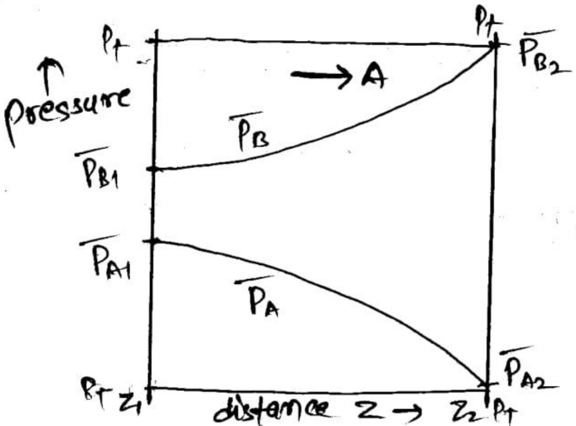



fig:- Diffusion of A through stagnant B

concⁿ. gradient of A $\frac{-d\bar{P}_A}{dz}$

if $z \uparrow$, $P_A \downarrow$

*2) Equimolar counter Diffusion :→ (EMCD) (steady-state)

Ex:- frequently pertains in distillation operation

$$\boxed{N_A = -N_B}$$

$$\boxed{N_A + N_B = 0}$$

* fundamental eq^n of MT $\quad N_A = J_A + \frac{C_A}{C}\cancelto{0}{(N_A + N_B)}$

$$\boxed{N_A = J_A} \quad \text{——— (1)}$$

$$N_A = -D_{AB}\frac{dC_A}{dz}$$

* Assume total concentration is constant $\quad x_A = \frac{C_A}{C}$

$$x_A C = C_A$$

$$dx_A \cdot c = dC_A \quad \text{——— (2)}$$

$$N_A = -D_{AB} \cdot C \frac{dx_A}{dz}$$

$$\int_{z_1}^{z_2} dz = -\frac{D_{AB} \cdot C}{N_A}\int_{x_{A1}}^{x_{A2}} dx_A$$

$$(z_2 - z_1) = -\frac{D_{AB} \cdot C}{N_A}(x_{A2} - x_{A1})$$

$$\boxed{N_A = \frac{D_{AB} \cdot C}{z}(x_{A1} - x_{A2})} \quad \text{——— (3)}$$

$$C_{A_1} = x_{A_1} C \; ; \; C_{A_2} = x_{A_2} C$$

$$\boxed{N_A = \frac{D_{AB}}{z}(C_{A1} - C_{A2})} \quad \text{——— (4)}$$

$$\left[ N_A = \frac{D_{AB} \cdot C}{z} (x_{A1} - x_{A2}) \right]$$

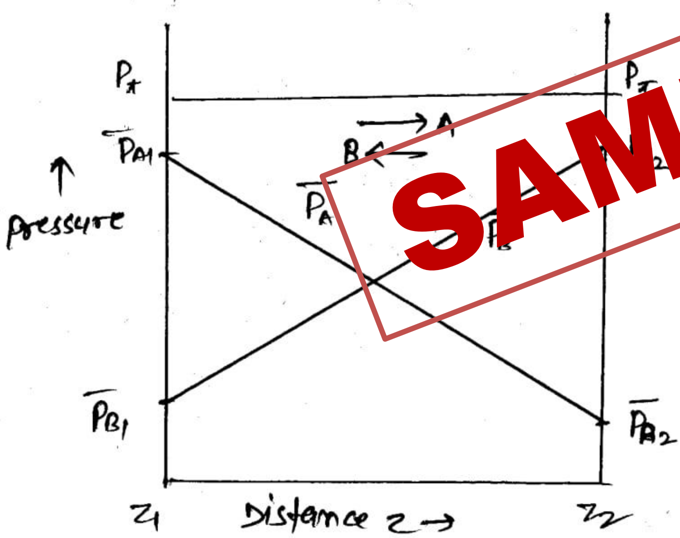
$$C \to P/RT$$

$$N_A = \frac{D_{AB} \cdot P}{RTz} (x_{A1} - x_{A2})$$

$$P_{A1} = x_{A1} P \; ; \; P_{A2} = P x_{A2}$$

Imp

$$\boxed{N_A = \frac{D_{AB}}{RTz} (P_{A1} - P_{A2})} \quad \text{———— (5)}$$






Equimolar counterdiffusion

starting point of A is $z_1$

starting point of B is $z_2$

A $\bullet\!\!\longrightarrow$ $\longleftarrow\!\!\bullet$ B

# Diffusion in Multicomponent mixtures :→ (steady-state)

* Effective diffusivity ($D_{A,m}$) :→ 
$$\left[ D_{A,m} = \frac{N_A - Y_A \sum_{i=A} N_i}{\sum_{i=A}^{n} \frac{1}{D_{Ai}} (Y_i N_A - Y_A N_i)} \right]$$

* if one component is stagnant

$$\boxed{D_{A,m} = \frac{1 - Y_A}{\sum_{i=B}^{n} \frac{Y_i}{D_{AB}}} = \frac{1}{\sum_{i=B}^{n} \frac{Y_i'}{D_{AB}}}}$$

where $Y_i'$ = mole fraction of component $i$ on an A free basis.

## * Mass Diffusivity (Diffusion coefficient) :- $D_{AB}$

# For gases ⇒ $\left\{ D_{AB} \propto \frac{T^{3/2}}{P} \right\}$ by kinetic theory of gases

where T = Absolute temperature (K)

Imp. [order of diffusivity of = $10^{-5}\ m^2/s$]

# for Liquids ⇒

$$\left[ D_{AB} \propto \frac{T\sqrt{M}}{\mu} \right]$$

where M → Molar mass of the component which is going to diffuse

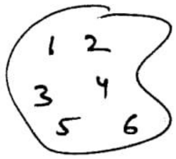


Imp. [order of diffusivity of = $10^{-13} - 10^{-15}\ m^2/s$]

## * Multicomponent diffusivity :-

The diffusivity of any component 1 in the mixture

$$D_{1-mix} = \frac{M_1 - y_1 \sum_{i=1}^{n} M_i}{\sum_{i=2}^{n} \frac{1}{D_{1-i}} (y_i M_1 + y_1 M_i)}$$



for 3 components

$$D_{1-mix} = \frac{M_1 - y_i (M_1 + M_2 + M_3)}{\frac{1}{D_{1-2}} (y_2 M_1 + y_1 M_2) + \frac{1}{D_{1-3}} (y_3 M_1 + y_1 M_3)}$$

* <u>special case</u> - If only one of the components from the mixture is diffusing and othere are non-diffusing.

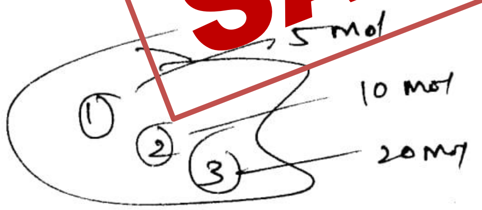
$$D_{1-Mix} = \frac{\cancel{M_1}(1-y_i)}{\sum_{i=2}^{n} \frac{1}{D_{1-i}} (y_i \cancel{M_1})} = \frac{1}{\sum_{i=2}^{n} \frac{1}{D_{1-i}} \left(\frac{y_i}{1-y_i}\right)}$$

$$D_{1-Mix} = \frac{1}{\sum_{i=2}^{n} \left(\frac{y_i'}{D_{1-i}}\right)}$$

where $y_i' = \dfrac{y_i}{1-y_i}$

$y_i'$ = is a mole fraction of species i except at the first component which is diffusing






$y_2' = \dfrac{10}{35}$

$y_3' = \dfrac{20}{35}$

Imp.

\# <u>variation of diffusivity with T and P</u> ⟹

* for <u>Gases</u> $\boxed{D_{AB} \propto T^{3/2} \propto \frac{1}{P}}$

* for <u>Liquids</u> $\boxed{D_{AB} \propto T}$ ; <u>pressure</u> has negligible effect

* for <u>solids</u> $\boxed{D_{AB} \propto T^{1/2}}$ ; <u>Pressure</u> has negligible effect

↳ diffusion in solids is called <u>knudsen diffusion</u>

* flux comparsion b/w ADNDB and EMCD :-

1) ADNDB $\boxed{N_A = J_A + x_A N}$

$J_A$ → contribution of a molecular diffusion

$x_A N$ → contribution of bulk flow

2) EMCD $\boxed{N_A = J_A}$ ; $(N = N_A + N_B = 0)$

so net flux w.r.t a stationary observer

$$\left[ (N_A)_{\text{Diffusion of A through non diffusion B}} > (N_A)_{\text{Equimolar counter diffusion}} \right]$$



Que 2) oxygen (A) is diffusing through metham (B) and Hydrogen (c) present in the volume ratio of 2 : 1 , Here both methane and hydrogen are non-diffusing. The diffusivity are given as

$$D_{O_2-H_2} = 6.99 \times 10^{-5}\ m^2/sec$$

$$D_{O_2-CH_4} = 1.86 \times 10^{-5}\ m^2/sec$$

Then the effective diffusivity of 'A' in mixture is $x \times 10^{-5}$ $m^2/sec$. then the value of $x$ is ________.

sol) Multicomponent diffusivity

$$\left[ D_{A,m} = \frac{1}{\frac{y_B}{D_{AB}} + \frac{y_C}{D_{AC}}} \right]$$

methane : Hydrogen
(B) (C)

$$y_B = \frac{2}{2+1} = \frac{2}{3}$$

$$y_C = \frac{1}{2+1} = \frac{1}{3}$$

$$D_{A,m} = \frac{1}{\frac{2/3}{D_{O_2-CH_4}} + \frac{1/3}{D_{O_2-H_2}}}$$

$$D_{Am} = \frac{1}{\frac{2/3}{(1.86 \times 10^{-5})} + \frac{1/3}{(6.99 \times 10^{-5})}}$$

$$D_{O_2,m} = \frac{1}{\frac{2/3}{(1.86\times10^{-5})} + \frac{1/3}{(6.99\times10^{-5})}} = 2.46\times10^{-5}$$

$D_{O_2,m} = 2.46\times10^{-5}\ m^2/sec$

$D_{O_2,m} = x \times 10^{-5}\ m^2/sec$

then $\boxed{x = 2.46}$ Answer

(similar Que in Gate-2016)

Que 3) What is mass flux of benzene through a layer of air having 10 mm thickness at 25°C at 200 kN/m². The partial pressure of benzene is 6 kN/m² at the left side of the layer and 1 kN/m² at the right side of the layer. The diffusivity of benzene in air is $4.4\times10^{-6}\ m^2/sec$

Sol → Case (I) ADNDB (Air (B) is insoluble in Benzene (A))

$$\left[ N_A = \frac{D_{AB}\cdot P}{RTz} \ln\left(\frac{P-P_{A2}}{P-P_{A1}}\right) \right]$$

Molecular weight of Benzene = 76 kg/kmol

given Data:- $D_{AB} = 4.4\times10^{-6}\ m^2/sec$

$P = 200\ kN/m^2 = 200\times10^3\ Pa$ $\quad (Pa \rightarrow N/m^2)$

$R = 8.314\ J/mol.K$

$T = 25°C + 273 = 298\ K$

$z = 10mm = 0.01\ m$

$P_{A1} = 6\ kPa$ $\quad$ $P_{A2} = 1\ kN/m^2$

$P_{A1} = 6\ kN/m^2 = 6\ kPa$

$P_{A2} = 1\ kN/m^2 = 1\ kPa$

$$\left[ N_A = \frac{D_{AB} \cdot P}{RTZ} \ln\left(\frac{P - P_{A2}}{P - P_{A1}}\right) \right]$$

$\left(1N \to 1\, kg \cdot m/s^2\right)$
$1J = N \cdot m$

$$N_A = \frac{(4.4 \times 10^{-6}) \times (200 \times 10^3)}{8.314 \times 298 \times 0.01} \ln\left(\frac{200 - 1}{200 - 6}\right)$$

$\frac{\cancel{m^2}}{s} \times \frac{kg \cdot m}{m^2 s^2} \times \frac{mol \cdot K}{\cancel{N \cdot m}} \times \frac{1}{K} \times \frac{1}{m}$ (N)

$$N_A = 9.0383 \times 10^{-4} \frac{mol}{m^2 \text{-} s}$$

Molar flux, $N_A = 9.038 \times 10^{-4} \frac{mol}{m^2 \text{-} s}$

Mass flux = Molar flux × mol. weight of Benzene

$$= 9.038 \times 10^{-4} \frac{mol}{m^2 \text{-} s} \times 78 \frac{kg}{kmol}$$

$(1\, kmol \to 1000\, mol)$

$$= \frac{9.038 \times 10^{-4} \times 78}{1000} \frac{kg}{m^2 \text{-} s}$$

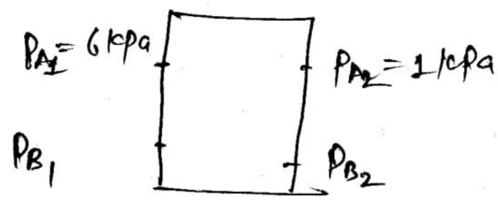
Mass flux $= 7.049 \times 10^{-5}\ kg/m^2\text{-}s$ Answer



(II) Method :

$$\left\{ N_A = \frac{D_{AB} \cdot P}{RTZ} \frac{(P_{A1} - P_{A2})}{P_{BLM}} \right\}$$

$$P_{BLM} = \frac{P_{B2} - P_{B1}}{\ln(P_{B2}/P_{B1})}$$




$P_T = 200\, kPa$

$P_{A1} + P_{B1} = P_T = 200$

$P_{B1} = 200 - P_{A1}$

$P_{B1} = 200 - 6 = 194\, kPa$

$P_{B1} = 194\, kPa$

$P_{A2} + P_{B2} = P_T = 200$

$P_{B2} = 200 - P_{A2}$

$P_{B2} = 200 - 1$

$P_{B2} = 199\, kPa$

$$\left[ P_{Blm} = \frac{P_{B_2} - P_{B_1}}{\ln\left(P_{B_2}/P_{B_1}\right)} \right] = \frac{199 - 194}{\ln\left(199/194\right)} = 196.489397 \text{ kPa}$$

$$\left[ N_A = \frac{D_{AB} \cdot P}{RTz} \; \frac{(P_{A_1} - P_{A_2})}{P_{Blm}} \right]$$

$$N_A = \frac{4.4 \times 10^{-6} \times 200 \times 10^3}{(8.314)(298)(0.010)} \; \frac{(\overset{6}{\cancel{199}} - \overset{1}{\cancel{194}})}{196.4893 \times 10^3}$$

$$N_A = 9.0383 \times 10^{-4} \text{ mol/m}^2\text{-s} \qquad \text{(molar flux)}$$

$$\text{mass flux} = 9.0383 \times 10^{-4} \times \frac{78}{1000} \; \frac{\text{mol}}{\text{m}^2\text{-s}} \; \frac{\text{kg}}{\text{mol}}$$

$$= 7.049 \times 10^{[illegible]} \text{ kg/m}^2\text{-s} \qquad \text{Answer}$$



(GATE-2011)

Que 4) Ammonia (component 1) is evaporated from a partially filled bottel into surrounding Air (component 2). The liquid level in the bottle and the concentration of ammonia at the top of the bottle are maintained constant. Assume that air in the bottle is stagnant. Which of the following option is correct

(A) ✓ $N_1 = \text{const}$, $N_2 = 0$ and $J_1 + J_2 = 0$

(B) $N_1 + N_2 = 0$ and $J_1 + J_2 = 0$

(C) $N_1 + N_2 = 0$ and $J_1 = \text{const.}$, $J_2 = 0$

(D) $N_1 = \text{const}$, $N_2 = 0$ and $J_1 = \text{const.}$, $J_2 = 0$

Sol: Ammonia (component 1),
Air (component 2)

Case (1) A diffusion in Non-diffusing B

so flux $N_A \neq 0, \quad N_B = 0$

(N) Net flux $\boxed{N_A = \text{constant}, \quad N_B = 0}$

so option (B) & (C) eliminate.

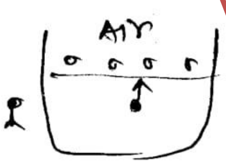
Now flux due to molecular diffusion

**ADNDB**

fundamental eq^n of MT

$$N_A = J_A + \frac{C_A}{C}(N_A + \cancel{N_B}^{\,0})$$

$N_A = \text{const}, \quad N_B = 0$




Air is stagnant $\boxed{J_1 = -J_2}$

$$\boxed{J_1 + J_2 = 0}$$

**EMCD**

$N_A = -N_B$

$$\boxed{N_A + N_B = 0}$$

$$N_A = J_A + \frac{C_A}{C}(\cancel{N_A + N_B}^{\,0})$$

$$N_B = J_A + \frac{C_B}{C_A}(\cancel{N_A + N_B}^{\,0})$$

$$N_A + N_B = J_A + J_B = 0$$

$$\boxed{J_A + J_B = 0}$$



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# for general case :-

$$N_A = J_A + \frac{C_A}{C}(N_A + N_B) \quad \text{--- (1)}$$

$$N_B = J_B + \frac{C_B}{C}(N_A + N_B) \quad \text{--- (2)}$$

eq (1) + (2) $\cancel{(N_A + N_B)} = (J_A + J_B) + \cancel{(N_A + N_B)}\cancel{\left(\frac{C_A + C_B}{C}\right)}$

If we assume overall concentration constant

$C = C_A + C_B$

$$\Rightarrow \boxed{J_A + J_B = 0}$$ always true for both cases

so correct option (A) $N_1 = \text{const}, \quad N_2 = 0$

$J_1 + J_2 = 0$

$$\left[Sh = c \cdot Re^{m} \cdot Sc^{n}\right]$$

where c, m and n can be found from experiment. Different values for different flow condition.

# Physical Meaning of Dimensionless Numbers :-

## 1) Sherwood Number (Sh) :-

Sherwood number is a measure of relative importance of convective mass transfer over the molecular mass transfer.

$$Sh = \frac{\text{convective mass transfer flux}}{\text{(Diffusive) molecular mass transfer through the stagnant film of thickness } l \text{ having same driving force}}$$

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$$Sh = \frac{k_c \Delta C}{\frac{D_{AB} \cdot \Delta C}{l}} = \frac{k_c \cdot l}{D_{AB}} \Rightarrow \boxed{Sh = \frac{k_c \cdot l}{D_{AB}}} ; \quad \frac{m/s \times m}{m^2/s}$$

or

$$Sh = \frac{k_G \cdot \cancel{(P_{A1} - P_{A2})}}{\frac{D_{AB} \cdot P}{RT\, l\, P_{BLM}} \cancel{(P_{A1} - P_{A2})}} = \frac{k_G\, RT \cdot l\, P_{BLM}}{D_{AB}\, P}$$

* for dilute solution, $\boxed{P_{BLM} \simeq P}$ $\Rightarrow Sh = \frac{k_G RT\, l}{D_{AB}}$

Relation b/w $k_G$ & $k_c$ ; Gas phase M.T.C

$$\boxed{\overset{\Delta y}{k_y} = \overset{\Delta P}{k_G P} = \overset{\Delta C}{k_c} \frac{P}{RT}} \Rightarrow k_G = \frac{k_c}{RT}$$

$$\Rightarrow \boxed{Sh = \frac{k_c\, l}{D_{AB}}}$$

$$\boxed{Sh = \frac{k_c \cdot l}{D_{AB}}}$$

where

$l$ = characterstic length

→ The role of sherwood number in mass transfer is analogous to the role of nusselt no. in heat transfer. $\left( Nu = \frac{h \cdot D}{k} \right)$

→ If $\boxed{Sh = 1}$ then mass transfer is by Molecular diffusion

[convective M.T = molecular M.T + Bulk]



2) Reynolds Number (Re) :-

It is a measure of relative importance of inertia force over viscous force

$$Re = \frac{\text{Inertia force}}{\text{viscous force}}$$

$$\boxed{Re = \frac{\rho u \cdot L_c}{\mu}}$$

3). Schmidt Number (Sc) :-

It is a measure of relative importance of momentum diffusivity over mass diffusivity.

$$Sc = \frac{\text{momentum diffusivity}}{\text{Mass diffusivity}} = \frac{\mu/\rho}{D_{AB}}$$

$$Sc = \frac{\mu}{\rho \cdot D_{AB}} \Rightarrow \boxed{Sc = \frac{\nu}{D_{AB}}} \; ; \; \nu = \frac{\mu}{\rho}.$$